“Dýnamis, the Babylonians, and *Theaetetus* 147c7–148d7”.

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ABSTRACT

Traditionally, the Greek mathematical term *dýnamis* is interpreted alternatingly as “square” and “root/side of square”. A survey of the usages of the term and of the related verb *dýnasthai* by Plato, Aristotle, and various mathematical authors including Eudemos/Hippocrates, Euclid, Archimedes, Hero, Diophantos, and Nicomachos, shows that all are compatible with a familiar concept of Babylonian mathematics, the square identified by (and hence with) its side. It turns out that a “geometers’ *dýnamis*” and a “calculators’ *dýnamis*” must be differentiated; that the technical usage for the former became fixed only around the mid-fourth century B.C.; and that it vanished except in specific connections and formulaic expressions by the third century.

Along with the conceptual congruity, Babylonian and Greek terms share a number of everyday connotations. This suggests that the Greek concept may have been inspired or borrowed from the Near East. This hypothesis can be neither proved nor disproved directly by the sources, but it is internally coherent and fruitful with regard to the existing material.

RESUMÉ

La tradition interprête alternativement le terme mathématique grec *dýnamis* comme “carré” et “racine carrée”. Un aperçu sur les modes d’emploi du terme grec chez Platon, Aristote, et chez un nombre de mathématiciens (dont Eudème/Hippocrate, Euclide, Archimède, Héron, Diophante, et Nicomaque) fait pourtant voir que l’on peut comprendre tous ces modes d’emplois à partir d’un concept familier aux mathématiques babyloniennes, à savoir le carré identifié par (et donc avec) son côté. Il s’ensuit aussi qu’il faut distinguer entre la “*dýnamis* des géomètres” et la “*dýnamis* des calculateurs”; que l’usage du premier ne devient fixe qu’au milieu du quatrième siècle avant J.-C; et qu’il disparaît du discours géométrique courant à partir du troisième siècle avant J.-C. et n’est conservé que dans des contextes spécifiques.

Le contenu conceptuel commun et l’existence de connotations secondaires partagées suggèrent la possibilité d’un emprunt du concept. Cette hypothèse ne se laisse ni prouver ni réfuter directement par les sources; elle résulte pourtant cohérente et féconde pour l’interprétation des documents existants.

Among the most debated single terms of ancient Greek mathematics is the word *dýnamis*[1], the basic everyday meaning of which is “power”, “might”, “strength”, “ability” etc. [GEL, 452[a-b]]. Responsible for this debate are first of all the paradoxical ways in which Plato uses the term in *Theaetetus*, especially because these ways appear to disagree with Euclid’s use of the term in the *Elements*.

The word is absent from Books I through IX of the *Elements*. In Book X, def. 2, however we read that straight lines (εὐθεῖαι) are “commensurable in respect of *dýnamis* (δυνάμει σύμμετροί) whenever the squares on them (τὰ ἀπ’ αὐτῶς τετράγωνα) are measured by the same area”. This indicates that *dýnamis* should be read as “square”, while raising the problem of why it is used instead of the current term *tetragon*.

In Plato’s *Theaetetus*, a “*dýnamis* of three feet” ([δύναμις] τρίπους) appears to be a square of the area three square feet (147d 3-4)[2]. A little bit later, however, *dýnamis* is the term chosen for certain *lines* (γραμμαὶ) – *viz*., lines which “square off” (τετραγωνίζειν) non-square numbers (anachronistically expressed, lines the lengths of which are surd). The latter use of the word has given rise to the other traditional interpretation of the word, as “side of square” or “square root” – eventually as “irrational square root”.

A third text has often been taken into account in these discussions. In Eudemos’ account of Hippocrates of Chios’ investigation of the lunes (as quoted by Simplicios [Thomas 1939, 238]) it is stated (in words which may perhaps go back to Hippocrates himself) that similar circular segments have the same ratios “as their bases in respect of *dýnamis*” (καὶ αἵ bάσεις αυτῶν δυνάμει), while circles have the same ratio “as the diameters in respect of *dýnamis*”. The Euclidean dative form *dynámei* is thus found (with approximately the same meaning) in a text dating back to the fourth or maybe even the fifth century.

FURTHER OCCURRENCES: THE EARLY EPOCH

In this paper, I intend to show that the apparently equivocal use of the term need not be considered equivocal after all, by pointing out an analogous conceptual structure in Babylonian mathematics. Before presenting this parallel I shall, however, give a more precise survey of the mathematical uses of the Greek term, in order to uncover more fully its uses and development.

There are, indeed, a number of less frequently discussed occurrences of the term and of the related verb *dýnasthai* (δύνασθαι; non-technical meaning “to be able/strong enough (to

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[1] Extensive references to the debate prior to the year 1975 will be found in [Burnyeat 1978]. Among later discussions of the term, [Knorr 1975], [Taisbak 1980] and [Taisbak 1982] should be mentioned.

[2] Burnyeat [1978, 492f] renders the whole passage 147c7 to 148d7 quoting John McDowell’s English translation, rendering δύναμις as “power”. In the Loeb edition, Fowler [1921] translates the term as “root”.

do something)", "to be worth", "to be able to produce", etc. – GEL, 451^b-452^a). As a preliminary (semantically uncommitted) translation integrating connotations of physical power as well as commercial value, I shall use "be worth" when discussing the mathematical uses of the verb. Instead of the expression "in respect of *dýnamis*" I shall mostly use the Greek dative *dynámei*.

The verb is used in close connection with the noun in the central *Theaetetus* passage (148a6-b2):

> THEAETETOS. We defined all the lines that square off equal-sided numbers on plane surfaces as lengths, and all the lines that square off oblong [*i.e.*, non-square – JH] numbers as *dynámeis*, since they aren't commensurable with the first sort in respect of length but only in respect of the plane figures which they are worth.

This translation reproduces McDowell as quoted by Burnyeat [1978, 493], with these exceptions: "*dynámeis*" is used instead of "powers"; "are worth" instead of "have the power to form"; and "in respect of length" instead of "in length", in order to render the parallel uses of the dative forms δυνάμει and μήκει. It can be seen that the lines which are labelled *dynámeis* "are worth" those squares of which they are the sides (anachronistically: The line of length √3 "is worth" the square of area 3).

In the Eudemos/Hippocrates fragment, the diameter d_1 of one circle is said to "be worth" the sextuple of another circle-diameter d_2 when it "is" its sextuple *dynámei*, i.e., when $d_1^2=6d_2^2$ (248^5 and 250^5 combined); the diameter of a circle, being the double of the radius "in length" (μήκει) is its quadruple *dynámei* (250^4). Furthermore, the two short sides in a right-angled triangle "are worth the same" (ἴσον) as the hypotenuse (250^1), while a line *a* is said to "be worth less" than two others *b* and *c* when $a^2<b^2+c^2$ (242^4).

In Aristotle's *De incessu animalium* 708^{b}33-709^{a}2, on the other hand, the hypotenuse of a right-angled triangle is said to "be worth" (*not* "worth the same as") the two other sides[3]; according to Heath ([1949, 284] against GEL 452^{a}44-45 following the Oxford translation), the same usage is meant in 709^{a}18-22. An identical formulation of the Pythagorean theorem is found in the pseudo-Aristotelian *De lineis insecabilibus* 970^{a}12-14.

In connection with a general discussion of "potency" and "potent" (δύναμις and δυνατός, respectively), Aristotle explains in *Metaphysica* 1019^{b}33-34 that the term *dýnamis* is used in geometry "by metaphor"; in 1046^{a}6-8 he explains the usage as due to "resemblance" (ὁμοιότης). An explanation of the concept as derived from Aristotelian (or older natural) philosophy should thus be excluded – even though a metaphor along the lines of "the square which a line is able to produce" would perhaps not be far from Aristotle's own understanding of the term[4].

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[3] It should be kept in mind that the Greek verb is transitive; "*x* being worth *Y*" is thus as different from "*x* being worth the same as *Y*" as "*x* loving *Y*" is from "*x* loving the same as *Y*" (jealousy apart).

[4] Formulations like the latter are found in various commentators from late antiquity (see Burnyeat [1978, 500 n. 34]. An explicit derivation from natural philosophy is considered "beyond doubt" by Bärthlein [1965, 45], who, substantiating his claim, mixes up lines and numbers in quite anachronistic ways.

The examples given so far demonstrate beyond a doubt that *dýnamis* and *dýnasthai* belong to accepted fourth- (and maybe fifth-) century geometrical parlance. They might also suggest that the use of *dýnamis* in *Theaetetus* as a designation for a line (be it a specific sort of line) is a Platonic hint of an idiosyncrasy of the young Theaetetos – as suggested by Burnyeat [1978, 496].

The first of these theses is confirmed by another Platonic passage, while the second is falsified (*pace* Burnyeat). *Politicus* 266a-b contains a pun on the word (already discussed by Burnyeat [1978, 496] and by Szabó [1969, 90]): Man, having the ability (*dýnamis*) to walk on two feet (being "two-feet in respect of ability"/δίπους δυνάμει) is identified with the diagonal [of the unit square], which is also "two-feet *dynámei*". Similarly, the swine, being four footed in respect of ability, is the "diagonal of the diagonal" (being four-feet *dynámei* it must be of length 2, and so be the diagonal of a square with side $\sqrt{2}$). We observe that the "human" diagonal is regarded in the second instance as something possessing itself a diagonal, i.e., as a square, in a way which defies both the interpretation of the *dýnamis* of a square pure and simple and the traditional alternative "side"/"square root".

Because Theaetetos and "the young Socrates" participate together in the dialogue as they do in *Theaetetus*, Burnyeat interprets the passage as another reference to Theaetetos' characteristic idiom. The pun is, however, put forward by the "Stranger from Elea", and furthermore with the words "since both of you are devoted to geometry". Had Plato wanted to hint at Theaetetos' own terminological contributions or habits he would hardly have chosen this way to express himself. Instead, the pun must be a play on the familiar and shared terminology of contemporary geometers of the period (or, rather, a terminology which a mid-fourth century philosopher would find natural in the mouth of a late fifth-century geometer).

FURTHER OCCURRENCES: THE EPOCH OF MATURITY

As is well known, almost all sources for the history of Greek mathematics date from the third century B.C. or later. Truly, in this age of maturity Greek mathematicians tended to make less use of the *dýnamis/dýnasthai-structure* than their forerunners appear to have done in the period from Hippocrates to Eudemos. Still, both terms occur a number of times in the great mathematical authors from Euclid onward, in ways which may serve to elucidate the terminology, showing continuity with earlier usages of varying character.

In *Data* 64, 65, and 67, Euclid speaks in the enunciations of the amount by which one side of a triangle "is worth" more or less than the other two sides, with the same meaning and in the same connection as Hippocrates/Eudemos. In the ensuing demonstrations, however, he only refers to "the tetragons on" the sides. The same thing happens in proposition 86. It appears as if the *dýnamis/dýnasthai* usage had been current at a time (fifth and fourth century) when certain theorems and standardized expressions were first formulated (the point in question here being the extended Pythagorean theorem), and that those formulations were handed down faithfully[5]. But the actual proofs of the *Data* were formulated in current words, speaking of tetragons and not *dynámei*.

Stronger evidence for a changing usage is seen in the *Elements*. Here the *dýnamis* is avoided even in the formulations of the theorems until Book X. So, the Pythagorean theorem, which both Eudemos/Hippocrates and the Aristotelian corpus refer to time and again in *dýnasthai*-dress, deals here with "the tetragons on" the sides (I.47). The same holds for XII.2, in which "circles are to each other as the tetragons on their diameters", whereas Eudemos/Hippocrates had spoken of the ratio "between the diameters *dynámei*".

In Books X and XIII we find the traditional usage – but only in definitions, in theorems, and in proofs referring to definitions or theorems or (in a few cases) summing up a result in formulaic language. During the free discursive argumentation on figures, all references are to "the tetragons on" the lines in question. X, def. 2, which was already quoted above, explains the formula "commensurability *dynámei*" of two straight lines as "commensurability of the tetragons on" the lines, and can thus be taken as a paradigm for the general relation between formulae and free speech.

The formulae which are used belong without exception to types with which we are already familiar from earlier sources. We find the counterposition of "commensurability in respect of length" (μήκει) and *dynámei* (e.g. X, def. 3); line *a* "being worth more" than line *b* (e.g. X.14), "being worth *n* times" b (e.g. XIII.2), or line *a* "being worth" lines *b* and *c* (e.g. XIII.10). Finally, a line may "be worth" an area (e.g. X.40) or a figure (e.g. XIII.1).

On the faith of Proclos, Archimedes is normally taken to have worked after Euclid. As observed by Schneider [1979, 61f n. 82] and Knorr [1978, 221], however, his works build on pre-Euclidean mathematics and not on the Euclidean *Elements*; as a witness of early terminology, he can thus be considered on a par with Euclid.

Archimedes' use of the *dýnamis/dýnasthai* terminology varies from work to work – a fact which was used by Knorr as supplementary evidence in his investigation of the relative chronology of the Archimedean corpus [1978, 264 n. 124a]. Most of the occurrences fall under the types also attested in Euclid: Ratio *dynámei* in contrast to ratio *simpliciter* or *mékei*; and a line "being worth" a rectangle or a plane figure. At times, however, a line "is worth the same" as a rectangle (e.g. *De sphaera et cylindro* I.29, 124^1). Furthermore, there seems to be a tendency (according to Knorr's relative chronology) for earlier works to use occasionally the idiom in free speech and for late works to restrict it to formulaic expressions and quotations of established theorems.

Like Euclid's work, the Archimedean corpus thus suggests that the *dýnamis/dýnasthai*-usage was being left behind in the free language of third-century geometers while being preserved (and still used) in a frozen state in formulaic expressions. This is further confirmed in Apollonios' *Conica*, with one qualification: Apollonios takes advantage of the possibilities of the terms to compress complicated expressions, creating formulae of his own (e.g. III.54, 440^{15}, where a ratio is composed from one ratio *dynámei* and another ordinary ratio between areas).

[5] Aujac [1984; 1984a] has investigated such word-by-word preservation of the phrasing of theorems, involving also Euclid and pre-Euclidean spherics.

Later geometers would still use the formulae but only by tradition. This is demonstrated by Pappos, in whose *Mathematical Collection* (along with some 20 correct quotations of the old formulations) the *dýnamis* and *tetragon* formulations of the *Data* are mixed up as "the *dynámeis* of the sides of the triangles" ($638^{11\text{-}13}$). Direct and indirect testimony is supplied by an anonymous 2nd century A.D. commentary to *Theaetetus* [Burnyeat 1978, 497]: It tells that "the ancients called tetragons *dynámeis*"; evidently, the readers are supposed not to know – and the commentator on his part seems not to know that the two terms though somehow semantically connected were used differently.

It is then no wonder that even Hero speaks of ratios *dynámei v. mékei* in *Metrica* I.19, 54^{18} – nor that a passage of I.34 (82^{28f}) appears to make a rectangle and not a line the subject of the verb *dýnasthai* (appears, since the passage is anyhow illegitimately elliptic and therefore possibly corrupt[6]). At other places, however, striking deviations from familiar expressions turn up. A passage in I.15 ($42^{22\text{-}25}$) runs "and take away from *dynámei* 121 *dynámei* 36, remainder *dynámei* 25, which is *mékei* 5". *Dynámei* 121 is thus simply $\sqrt{121}=11$, which in a more traditional formulation might appear as "that which *dynámei* is 121", corresponding also to the expression "ΒΘ *dynámei* 180" found three lines above (freely to be interpreted $\text{ΒΘ}^2=180$ or $\text{ΒΘ}=\sqrt{180}$). But the phrase in lines 22-24 contains none of those articles and relative pronouns which in normal Greek mathematical texts indicate elided words. *Dynámei N* is simply used for $\sqrt{N}$.

If we go to I.17, 48^{5f}, on the other hand, "the <ratio> of the *dýnamis* of the <tetragon> on ΒΓ to the <tetragon> on ΒΓ upon the <tetragon> on ΑΔ" designates the ratio of ΒΓ^4 to $\text{ΒΓ}^2 \cdot \text{ΑΔ}^2$. *Dýnamis* N is thus N^2. So, the Platonic ambiguity between "square" and "square root" turns up again in this rather late and very un-Platonic text (though grammatically distinguished as it should be in an efficient technical terminology).

THE "CALCULATOR'S DYNAMIS"

The ΒΓ^4 of *Metrica* I.17 is also spoken of as "the *dynamodýnamis* upon ΒΓ" (48^{21}), Diophantos' term for the fourth power. It might therefore seem that the numerically oriented mathematicians of later antiquity merely embraced a traditional geometrical concept and shaped it for their own purpose. I shall call this concept (that of Hippocrates, Euclid, Archimedes,

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[6] Hero cites Archimedes, *De conoidibus et sphaeroidibus*, for the statement that "the$^{N/A}$ <rectangle> under the axes [of an ellipse] is worth the circle^{A} equal^{A} to the ellipse" [^{N}=nominative case ending, ^{A}=accusative], but afterwards uses the correct theorem that the product of the axes equals the *square* of the diameter of the circle in question. In a footnote, Heiberg proposes the correction "... is worth <the diameter^{A}> of the circle^{G} equal^{G} ..." [^{G}=genitive], which would still be irregular; the emendation "... is worth <the diameter^{N}> of the circle^{G} equal^{G} ...", however, would put everything straight, apart from a legitimate though rather unusual inversion.

Apollonios and Pappos) the "geometers' *dýnamis*", in agreement with the passages from *Metaphysica* and *Politicus* quoted above. More likely, however, the similarities between Plato's and Hero's texts should be explained with reference to an old, related but distinct "calculators' *dýnamis*". To this point I shall return; for the moment I shall only argue for the existence of the entity in question.

It turns up rather explicitly in Plato's *Republica* 587d, during the discussion of the distance between the tyrant's phantasmagoric pleasure and real pleasure, which, when regarded as "number of the length" (τοῦ μήκους ἀριθμός), is argued by Socrates to be the "plane number" $3 \cdot 3=9$. It is then "clear, in truth, how great a distance it is removed according to *dýnamis* and third increase" (κατὰ δύναμιν καὶ τρίτην αὔχην) – a statement upon which Glaucon comments: "clear at least to the calculator" (δῆlος τῷ γε lογιστικῷ). In this gently ironic portrait of his brother[7] Plato evidently supposes that the mathematically illiterate will have known the word *dýnamis* as belonging to the field of practical calculation (logistics) rather than to that of theoretical geometry. Furthermore, logistics is supposed by Socrates' remark to deal with three different numerical manifestations of one and the same entity, as "number of the length", *dýnamis*, and "third increase". Kindly enough, Plato tells us that these are not just the "linear", "square", and "cube numbers" known from Greek theoretical arithmetic (and from *Theaetetus*), the "number of the length" being already a square number; they have to correspond to the first, second and third power of the entity.

Presumably, the "calculators' *dýnamis*" is also mentioned in *Timaeus* 31c-32a[8]. At most, however, this passage provides us with the extra information that the terminology for the "third power" was vacillating. More interesting as an elucidation of the *Republica* passage and of the "calculators' dýnamis" are the terms used in Diophantos' *Arithmetica*. As he explains in his foreword, Diophantos speaks of square and cube numbers as "tetragons" (τετράγωνοι) and "cubes" (κύbοι), respectively (2^{18-22}). *In agreement with general convention*, however, the second and third power of the unknown number (the ἀριθμός) are spoken of as *dýnamis* (δύναμις, abbreviated Δ^{Y}) and cube (κύbος/K^{Y}) (4^{15-17})[9]. Now, it is known that part of Diophantos'

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[7] The reading of the passage as benign irony is supported by the similar portrait of the *jeunessse-doré*-attitudes of the other brother Adeimantos in 420a.

[8] Souilhé [1919, 124] reads the passage differently, equating δύναμις with "force" and ὄκος with "masse". This is not very plausible in view of the context.

This passage exhausts the number of mathematical occurrences of the *dýnamis* in the Platonic corpus, together with another passage in *Timaeus* (54b), where in the triangle obtained by bisection of the equilateral triangle one side is said to be the triple of the other "according to *dýnamis*" (κατὰ δύναμιν). (I disregard a possible hint in the notoriously obscure *Republica* 546b, and the occurrences in the pseudo-Platonic *Epinomis*).

[9] As pointed out by Rashed [1984, 113], the term *dýnamis* is introduced at an earlier stage than unknown numbers. Only by saying that "it has been approved" (ἐδοκιμάσθη) that in this form the square of numbers becomes one of the "elements of arithmetical theory" (στοιχεῖοσ τῆς ἀριθμετικῆς θεωρίασ), does Diophantos make clear that he is already here aiming at the only actual use of the term later on, *viz.*, as a designation for the square of the unknown ἀριθμός.

algebraic formalism is taken from earlier Greek calculators: The abbreviation ϛ for the ἀριθμός is used in a c. 1st century (A.D.) papyrus (see [Robbins 1929] and [Vogel 1930]), and the term δυναμοδύναμις for the fourth power was used during the same century by Hero (cf. above). Furthermore, part of Diophantos' material (I.xvi-xix, xxii-xxv) is borrowed from traditions of recreational mathematics ("purchase of a horse", "finding a purse", etc.; see [Tropfke/Vogel 1980, 606-613]) which already in Plato's time had given rise to theoretical treatment ("Thymarides flower"; see [Heath 1921, 94ff]). Since the distinction made between *square number* and *dýnamis* coincides with that made in *Republica* 587d, it appears reasonable to assume that even this is due to continuity, and that the Diophantos' "general convention" followed the old calculators known to Glaucon in its specific use of *dýnamis*[10].

If this is so, "geometers'" and "calculators' *dýnamis*" are of course related but yet different concepts, and one must be assumed to derive from the other. For the moment, we will have to leave open the question of the direction of influence, and return our attention to the geometers' concept, which is better documented in the sources.

INTERPRETING (I)

The difficulty of explaining *dýnamis* plainly as another name either for *tetragon* or for *side* is as evident as the difficulty of explaining away the evidence in favor of the rival explanation. Instead, two new interpretations (both involving centrally the verb *dýnasthai*) have been proposed by Szabó and Taisbak.

Taisbak [1980; summarized in 1982, 72-76] proposed a reading of *dýnasthai* as "to master", in the sense that a line "masters" that two-dimensional extension which it is able to cover by a square; this *extension* should be understood as an entity different from both the square as a *geometrical figure* and from its area regarded as a *number* resulting from mensuration. In its origin, *dýnamis* should then be a term for the extension. For later times, Taisbak proposed a reduction to an ill-understood rudiment. The use of the term for a line should result from informal speaking among mathematicians.

Szabó's explanation [1969, 46f; reworked 1986] built on the well-documented use of

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At the same time, he notes that he is following a general convention from a discipline of "arithmetical theory" which is neither Euclidean nor Neopythagorean (Nicomachos uses the term quite differently, as we shall see). Only Diophantos' own brand of arithmetic seems to be left, i.e., *algebra.*

[10] Few instances of ancient second-degree "algebra" below the level of Diophantos have survived in sources from classical antiquity. Some, however, can be found scattered throughout surveyors' and related texts. E.g., in the *Geometrica* ascribed to Hero, xxi.9-10 (380$^{15\text{-}31}$), the dimensions of a circle are found from the sum of diameter, perimeter, and area, while the Roman agrimensor Nipsus (2nd c. A.D?) treats the problem of a right-angled triangle with known hypotenuse and area in his *Podismus* (297f). We can hence be sure that basic second-degree "algebra" was indeed known to the ancient practitioners.

dýnasthai as "being worth" in a real commercial sense ("the *shekel* is worth 7 obols"). This is supposed to have inspired a use expressing the notion that *a square* is equal to *some other surface* (a rectangle or a sum of squares); for some reason ("irgendwie" [1986, 359]), the expression involves the side of the square as the subject, and not the square itself. Formally, a *dýnamis* should be a line; in reality, however, it should denote the square constructed upon the line, but only on condition that this square is equal to another surface.

In order to underpin his interpretation, Szabó claimed that the κατὰ δύναμιν usage of the passages from *Republica* and *Timaeus* (in fact the earliest certain appearances of the mathematical *dýnamis*) is derivative, while the dative *dynámei* used from the late Platonic dialogues onward reflects the original thinking. Even if this hypothesis is granted, the rather loose language of the remaining pre-Euclidean sources is problematic for a strict reading of Szabó's thesis – a line being sometimes worth other lines, sometimes "the same" as other lines, etc. If the thesis is read more loosely than originally intended, however, as informal speaking, neither the early Platonic occurrences nor the lax formulations are serious challenges; interpreted like this, on the other hand, the explanation comes close to Taisbak's.

Before considering either of these positions, I shall step outside the circle of Greek language and culture.

A BABYLONIAN PARALLEL

To a historian of Babylonian mathematics, the apparent ambiguity between "square" and "square root" has a familiar ring. Both the basic Old Babylonian term for a geometric square (*mitḫartum*) and the Sumerogram normally translated as "square root" (í b - s i$_8$) appear (when translated into modern terminology and concepts) to designate alternatingly the square and its side. The semantic basis of í b - s i$_8$ is *equality* (*viz.*, equality of the sides of a square), whereas that of *mitḫartum* is the confrontation of equivalents (still as sides of a square). Interestingly, the Babylonian term for "countervalue" or "commercial rate" (*maḫirum*) derives from the same root as *mitḫartum*, *viz.*, from *maḫārum*, "to stand up against, to encounter, to receive [an antagonist, an equivalent, a peer]". So, the linking of "square", "side of square", "commercial rate", "equivalence", and "confrontation of force", so puzzling in Greek mathematics, is shared with the mathematics of the old eastern neighbor. Could it be that the Greek term translates a borrowed technical concept, using a Greek term possessing the same connotational range as the original Semitic term[11]? And could a possible borrowing, or simply the conceptual parallel,

[11] Next to nothing is known about the transmission of Babylonian mathematics after the end of the Old Babylonian period (c. 1600 B.C.), but that transmission took place is sure. As I have shown in my [1986, 457-468], a 12th-century Latin translation from the Arabic follows Old Babylonian ways down to the choice of grammatical forms. That the Greek calculators owed part of their technique to the Near East is also apparent from the name of their favorite instrument, the ἄβαξ, the [dust] abacus, which is borrowed from western Semitic **ʾbq**, "light dust" (the root is absent in Babylonian). Since finally the term *maḫirum* is testified in Hebrew

help us understand the nuances of the Greek term?

Since our earliest sources (be it Plato or the Eudemos/Hippocrates fragment) use the *dýnamis*-terminology in developed form, the original idea behind it cannot be established beyond doubt, and conceptual and terminological diffusion (from Babylonia or, indeed, from anywhere) can be neither proved nor ruled out as a possibility. The answer to the first question is an uninteresting "yes – anything *could* be". For the time being, the hypothesis can only be tested for plausibility and fruitfulness, the former depending largely on the latter, i.e., on the answer to our second question. We shall therefore need a closer look at the Babylonian concepts.

According to its derivation and to cognate terms, *mitḫartum* designates an entity arising from the confrontation of equivalents (the confrontation of the line and its *meḫrum* or "counterpart" – another derivative from the same root). A number of texts show that the *mitḫartum*, when a number is ascribed to it, *is* the length of the side and *possesses* an area[12]. No single text can be found where the square is identified with its area, as *we* would tend to do, and as is inherent in the Euclidean tetragon as a "figure" (σχῆμα), i.e., as something which is "encompassed by some boundary or boundaries" (ὑπό τινος ἤ τινως ὅρως περιεχόμενον) (*Elements* I, def. 22 and 14). On the other hand, other evidence shows beyond a doubt that the *mitḫartum is* a geometrical square and not a mere line adjacent to a square – e.g., BM 15285 [MKT I, 137f], where the squares are *drawn*.

This may seem strange to us. From a culturally neutral standpoint, however, our own ways are equally strange. Why should a complex geometrical configuration – four equal lines at right angles delimiting a plane surface – be considered identical with the measure of the plane surface, rather than with the measure of one of the lines? Once the configuration is given, one parametrization is as good as the other. So, the ambiguity of the *mitḫartum* concept vanishes: It is not alternatingly square and square root, but simply the figure identified by – and hence *with* – its side.

The case of íb-si$_8$ is similar. Etymologically and in most occurrences the term is a verb. A phrase like "81-e 9 íb-si$_8$" must apparently be read as "81 makes 9 equal-[sided]"[13]. In some occurrences, the term is used as a noun, related to *mitḫartum*, i.e., as a square figure parametrized by the length of its side – at times when the side of a square of known area is asked for, but occasionally as a description of the geometrical configuration itself. In some

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in the related form ***m^{e}ḥīr***, a Western Semitic (Phoenician?) contact is no less linguistically possible than direct Babylonian influence.

Without taking Proclos' *Commentary* more seriously than it deserves, we may also remember his ascription in 65^5 of "accurate investigation of numbers" (τῶς ἀριθμῶν ἀκριβὴς γνῶσις) to the Phoenicians, which he derives from the needs of logistics.

[12] E.g., BM 13901, *passim* [MKT III, 1-5]. The first problem can be translated: "I have added the area and my *mitḫartum*, it is $^3/_4$". The solution states that the *mitḫartum*, the square identified with its side, is $^1/_2$.

[13] This follows both from the Sumerian ergative suffix -e and from interrogative variants of the phrase showing 9 to be an accusative. Exemplifications can be traced through the glossaries of MKT.

instances, finally, the term occurs as a verb denoting the creation from a length of the corresponding quadratic figure (but *not* its area)[14]♦. Once again, the square is considered under the aspect of a figure made up of equal sides, not as a plane surface surrounded by such sides.

INTERPRETING (II)

With this in mind we shall return to the Greek material – first to the concepts "commensurable in respect of length" (μήκει σύμμετροί) and "commensurable in respect of *dýnamis*" (δυνάμει σύμμετροί) from *Elements* X, def. 2-3. Two straight lines (εὐθεῖαι [γραμμαὶ]) are commensurable "in respect of length" if they have a common measure when each is regarded without sophistication as a length – a μήκος. They are commensurable "in respect of *dýnamis*" when the tetragons on them have a common measure – that is, when *the two lines themselves* are *commensurable if regarded in the Babylonian way*, as representing squares. The common grammatical form (the dative) of μήκει and δυνάμει suggests that the two terms should stand in the same relation to the straight lines; since the line *can* indubitably be apprehended as a length, it should also be possible to apprehend it as a *dýnamis* (and it *should* be seen so in "commensurability in respect of *dýnamis*"). But the parallel leads still further. Since in the former case the lengths themselves have the common measure, in the latter case the *dynámeis* must be the things measured (remember that the Greek measuring procedure is a process of covering or taking away, cf. the *anthyphairesis*). The *dýnamis* can hence hardly be anything but a *mitḫartum*, a square identified with its side (but still of course possessing an area to which a measuring number can be ascribed). Otherwise expressed, the *dýnamis* is a *line seen under the aspect of square.*

If instead of commensurability we had looked at ratio *dynámei* and *mékei*, as known from Archimedes, the same arguments could have been developed. In both cases it becomes evident why we never find expressions like "commensurability in respect of tetragon" or "ratio in respect of tetragon": Tetragons themselves are commensurable (if they are) and in possession of a mutual ratio – they are not aspects of a line. The absence of such expressions will also follow from Taisbak's interpretation of the term; it is, however, somewhat enigmatic if "*dýnamis*" is believed to be nothing but another word for "tetragon", Why, in fact, should *Elements* XII.2 when reformulating the Hippocratean theorem that circles have the same ratio "as their diameters *dynámei*" also change the grammatical construction if it had been meaningful to speak of ratios τετράγωνῳ? Truly, grammatical habits might have changed over the centuries, but this would then affect both terms had they really been synonyms (as, in fact, we see in Pappos' late mix-up).

If we turn to *Theaetetus*, the first use of *dýnamis* as a "square of three [square] feet" is of course in harmony with the interpretation of the term as a *mitḫartum* – τρίπους, "of three

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[14] A full documentation of the varying uses of íb-si$_8$ would lead too far astray. It belongs with a larger investigation of Babylonian "algebra" (work in progress; preliminary report in [H¢yrup 1984], final to appear in [H¢yrup 1990]).

feet", is an adjective and hence not necessarily to be regarded as an identity. The later passage, in which the young Theaetetos introduces his definition distinguishing two sorts of lines (γραμμαὶ), is more interesting: on one hand, a line which can be "spoken of" as a length, i.e., a line the length of which can be measured by a rational number which can be used as its name, is called a "length", a μῆκος. On the other hand a line which can only be "spoken of", i.e., be given a numerical name, when regarded under its aspect of *dýnamis*; such a line is called a *dýnamis* (it will be remembered that the Greek term which translates as "rational" is ῥητός, meaning "which can be spoken").

According to the *mitḫartum* interpretation, the definitions introduced by Theaetetos are no longer shocking, clumsy, or childish, as they have been regarded by various authors. Theaetetos does not call a square root a square, or anything like that. Truly, any line can in advance be regarded as a *dýnamis*, and Theaetetos restricts the use of the term to such lines which in a certain sense are *only* to be spoken of as *dynámeis*. This is, however, a precise analogue of another well-known Greek dichotomy: Some numbers are "square numbers": They can be "engendered as equal times equal" (ἴσον ισάκις γίγνεσθαι), i.e., produced as the product of two equal factors. In principle, a "square number" is also "oblong" – it can be produced as the product of unequal factors: $4{\cdot}4=8{\cdot}2$; $3{\cdot}3=9{\cdot}1$. The name "oblong number" (ἀριθμός προμήκης) is, however, reserved for such numbers which are *only* oblong, i.e., to non-square numbers. This delimitation is introduced by Theaetetos in the same dialogue just before the "shocking" definitions of "length" and "*dýnamis*" (147e9-148a4), and nobody has ever been shocked. Yet, according to the *mitḫartum* interpretation, the logic of the two definitions is strictly the same. No puzzles are left. The passages from *Theaetetus*, as well as the entire material on the "geometers' *dýnamis*", fit the interpretation of the *dýnamis* as a concept of the same structure as the Babylonian conceptualization of the square.

As already stated, the linking between *dýnamis*, commercial worth, and confrontation of force is a feature shared with the Babylonian *mitḫartum*. No Babylonian mathematical term equivalent to *dýnasthai* exists, however. Nor does there appear to be in Babylonian mathematics any concept or procedure which necessitates such a word. So, even if the *dýnamis* may be imported or inspired from Babylonia, the term *dýnasthai* appears to be a genuine Greek development due to the integration of the *dýnamis* concept into the theoretical structure of Greek geometry. We see in *Theaetetus* 148b2 a possible way for such a development, when Plato speaks of "the plane figures" which the lines *dýnantai*, i.e., "have in their power to form when seen *dynámei*" or "are worth" under the same aspect. This could also be the metaphorical sense of which Aristotle speaks in *Metaphysica* 1019^{b}33f, and it suggests that the Greeks may have conceptualized the term in Taisbak's manner in the mid-fourth century (and maybe earlier), independently of its origin. This, in connection with the verb's connotations of equivalence and being worth, could then easily lead to the general loose usage in which lines or surfaces (Hero!) can be said to *dýnasthai* other lines or surfaces, but where in all cases the equality involved is one of surfaces, not of lengths.

On the other hand, the *dýnamis* might also stand for a *mitḫartum*-like concept without having been borrowed at the conceptual level. Both concepts could have developed independently on the basis of analogous or shared measuring *practices*[15]. In this case, the shared secondary connotations of the two terms must be considered accidental (which, given the connotative richness of both languages, could easily have happened).

"CALCULATORS' DYNAMIS" REVISITED

So, if we restrict our reflections to the "geometers' *dýnamis*", conceptual borrowing and independent development of analogous conceptualizations of the square figure are equally good causal explanations of the apparent *mitḫartum*-structure of the Greek concept. This, however, brings us to the question of the "calculators' *dýnamis*". If, as was argued, Greek calculators may plausibly have been in possession of second-degree algebra showing terminological continuity up to Diophantos, it can hardly have been an indigenous development; it would have been inspired (or, more probably, imported) from some Middle Eastern algebra descending from the Old Babylonian tradition. Now, I have shown elsewhere that Old Babylonian "algebra" cannot have been arithmetical, i.e., conceptualized as dealing with unknown numbers organized by means of numerical operations[16]. Instead it appears to have been organized on the basis of "naive", non-deductive geometry, of a sort related to that used by al-Khwārizmī in his *Algebra* to justify the standard algorithms used to solve the basic mixed second-degree equations (see [Rosen 1831, 13-21], or one of the published Medieval Latin translations, e.g., [Hughes 1986, 236-241]), but of course without his Greek-type letter symbolism. Since the Arabic treatise mentioned in note 11 was of a similar sort, a descendant which inspired Greek calculators can hardly have been much different. Even early Greek "calculator-algebra" will consequently have dealt with "real" lines and squares, not with sums and products of pure numbers[17]. Truly, the "real" lines and squares may have been rows and patterns of pebbles on an abacus-board, rather than the continuous lines of a drawing – cf. below.

At the same time, the branch of Old Babylonian mathematics in which *mitḫartum* and í b - s i 8 occur most frequently is the so-called "algebra". So, if a conceptual import into Greece

[15] I am grateful to Professor Tilman Krischer of the Freie Universität Berlin for pointing out the importance of this possibility in his comments on an earlier version of the present paper.

[16] Once more, documentation would lead too far astray – cf. note 14 above. The simplest part of the evidence comes from an analysis of the terminological structure of the texts. Two different "additive" operations are kept strictly apart in a way which has no meaning in an arithmetical interpretation, i.e., if the terms were synonyms for the one and only numerical addition. Similarly, two different "subtractions" and four different "multiplications" are distinguished.

[17] If we take Plato's testimony at its words, it suggests the same. The third power was spoken of as the "third increase", which fits well with a spatial conceptualization but rather poorly with an arithmetical representation before the introduction of exponential symbolism or spatial representation. Arithmetically, we would have the number itself, the increase (i.e., the second power), and the second increase, i.e., our third power.

has indeed taken place, the plausible channel is “calculator-algebra” rather than theoretical geometry. This would make the “calculators’ *dýnamis*” the primary concept from which the “geometers’ *dýnamis*” would be derived.

Hero’s curious phraseology (“*dynámei* 25, which is *mékei* 5” – cf. above) might then belong rather with his calculator than with his Archimedean affiliation. It belongs indeed with a numerical calculation. As in *Republica* 587d, the same concrete entity is represented by several numbers; and as in the second passage from *Theaetetus*, the mathematics of the passage suggests the translation “root”. If the segregation of a geometrical *dýnamis* was only taking place during Plato’s (and Theaetetos’) youth, these specific parallels between Plato and Hero are probably manifestations of the closeness of both to calculators’ usage.

If, on the other hand, the *dýnamis*-concept was indigenously developed, we would rather expect its origin to belong with geometry and mensuration. This would make the “calculators’ *dýnamis*” a metaphor, and suggest that, in spite of its dependence on pre-scientific sources and methods, logistics had already come under the sway of scientific mathematics in respect of metaphorics and conceptualizations around 400 B.C. If one on the balance between references to logistics and to the purer branches of mathematics in the earlier part of the Platonic corpus (including *Republica* and *Timaeus*), this seems highly improbable.

THE DYNAMIS OF FIGURATE NUMBERS

Furthermore, an origin of our term in logistics will also fit its use in the “Pythagorean” theory of figurate numbers better than an origin in theoretical geometry. Here, indeed, the word turns up in a way which could well be related to its use in a “pebble-algebra” but not to its geometrical function.

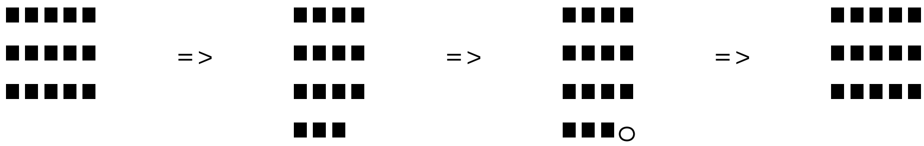
By “pebble-algebra” I refer to a possible representation of a second-degree “algebra” in Babylonian style by means of pebbles on the abacus board. Indeed, who says “calculator” in a Greek context says “pebble” or ψῆφος – the main tool of the calculator being the abacus with appurtenant pebble calculi. It is also a well-established fact that the “doctrine of odd and even”, as well as the whole theory of figurate numbers, grew out of the patterns in which pebbles could be arranged (cf. [Lefèvre 1981]). It is therefore near at hand to assume that *if* some calculator algebra was in use in classical Greece it was performed (exclusively or occasionally) with pebbles on the abacus board[18].

This observation is interesting for several reasons. *Firstly*, the interest in figurate numbers (including the “square” and “oblong” numbers spoken of by Theaetetos) ceases to be the result of some play with abacus pebbles irrelevant to their normal use. Square, gnomonic and oblong

[18] Since the abacus appears first to have been borrowed in the form of a dust abacus from the Near East (cf. above, note 11), and since this device was used for geometric drawings throughout antiquity, occasional use of real drawings on a dustboard is also a possibility and in fact appears to fit Nipsus’ problem (see note 10) better than pebble manipulation.

numbers occur naturally as soon as one tries to represent a mixed second-degree problem on the board. So, e.g., the problem $x+y=8$, $x \cdot y=15$ is represented and solved thus:

A ***B*** ***C*** ***D***



The virtual starting point for the analytical procedure is a pattern of 15 pebbles (*A*), whose length and width taken separately are unknown, whereas the sum of the length and the width is known to be *8*. In the real process of solution we therefore start by laying out a gnomon with *8/2=4* pebbles in each leg, and fill out the inside until all 15 pebbles have been used (*B*). This shows that [a square of $1 \cdot 1=$] *1* pebble is lacking in order to complete the square (*C*), and that hence *1* row has to be moved from bottom to the right in order to actualize the virtual rectangle (*D*)[19].

Apart from the occurrence of oblong, gnomonic and square numbers (all basic entities in the theory of figurate numbers) we see that one of the basic theorems of the theory follows immediately from the procedure – *viz.*, that the sum of the first *n* odd numbers equals n^2. Even the triangular numbers and the theorem that the sum of two consecutive triangular numbers is a square number are seen on the figure, although these observations play no role in the process. As soon as one starts reflecting theoretically on the patterns, triangular numbers and their properties, as well as those of the gnomonic, square and oblong numbers, turn up as obvious questions[20]; the theory of figurate numbers emerges as a *theory dealing with the general properties of existent tools and practices* instead of being an idle play picked up from nowhere.

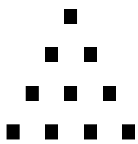
Secondly, an astonishing use of the term *dýnamis* in Pythagorean or Neopythagorean arithmetic becomes meaningful. In configuration *C*, the *mitḫartum-dýnamis* is evidently *4*. This is the line which "squares off" the complete pattern, in Theaetetos' words. Now, the term turns up in Nicomachos' *Introduction to Arithmetic* in a way which could easily be explained as a generalization of this usage but which is otherwise anomalous. If we look at configuration *A*, we see the number *15* being arranged in thirds – according to Nicomachos in parts which "by name" (ὀνοματί) are *3* and *dynámei* (or κατὰ δύναμιν – both forms are used) are *5* (see, e.g., I.viii.7, 16[1]]). This is no far-fetched transfer of the meaning in *C*, even though contact

[19] If the problem had been $x-y=2$, $x \cdot y=15$, we would start step *B* with the inner gnomon, the one with legs containing 2 pebbles, and add new layers at the outside. Apart from that, the same configurations would have to be used. Odd values of $x \pm y$, on the other hand, require further refinement.

[20] In his investigation of the prehistory of incommensurability, Knorr [1975, 142ff] comes to similar pebble-configurations and conclusions from another angle and deals with the matter in much more detail.

with the geometrical meaning is lost.

Other 1st or 2nd century (A.D.) doxographic sources suggest that the usage is not a Nicomachean idiosyncrasy. They concern one of the central Pythagorean concepts, the *tetractys* or decade drawn up as a triangular number:



According to Aëtius (*Placita* I.3.8), the Pythagoreans "declare ... that the *dýnamis* of ten is in four, and in the tetrad" (τῶν δέκα ...φησίν, ἐ δύναμις ἐστιν ἐν τοῖς τέσσαρσι καὶ τῆι τετράδι) (Fragment 58 B 15 [Diels/Kranz 1951 I, 544[1]]). Taken in itself this phrase is ambiguous, and could well mean that the power of the magical number 10 resides in its possible triangular arrangement as *tetractys*. Hierocles, however, is more explicit in a commentary to supposedly early Pythagorean writings, stating that "the *dýnamis* of the decade is the tetrad" (τῆς δε δεκάδος δύναμις ἐ τετράς) ([Mullach 1875 I, 464B], quoted from [Souilhé 1919, 23]). So, these two doxographers (who will hardly be suspected of innovative mathematical terminology) appear to refer to a generalization of the concept of *dýnamis* different from but very close to that of Nicomachos: Once more, the "base" of a non-square figurate number is taken as its characteristic parameter and given the name belonging to the same parameter in the case of a square figurate number.

FURTHER OBSERVATIONS

Can we get any nearer to the process, or has the meager material now been exhausted? We can in fact squeeze the sources harder, observing that the two "intermediate" Platonic dialogues contain the expression κατὰ δύναμιν, whereas the late dialogues (*Theaetetus, Politicus*) as well as all other authors (except the non-geometrical Nicomachos) invariably use the simple dative *dynámei*. This suggests that the technical use of the term was only crystallizing in Plato's later years, around the mid-fourth century; by then, on the other hand, a fully technical "geometers' *dýnamis*" *was* crystallizing.

Firstly, this observation makes it seem highly doubtful that Hippocrates' own words are rendered exactly in the Eudemos fragment, which agrees so perfectly with the style of late Platonic, Aristotelian, and Archimedean occurrences[21]. The fragment seems rather to contain Eudemos' reformulations in his own phrase structures of Hippocrates' ideas, concepts and basic terms (including probably some forms of *dýnamis* and *dýnasthai*). *This conclusion is independent of all other hypotheses on the meaning and origin of our terms.*

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[21] The same doubt as to the literal precision of Eudemos' quotation was recently formulated by Knorr [1986, 38f] on the basis of other evidence.

Secondly, cautious assumptions on the temporal distance between the introduction of a mathematical terminology and its crystallization in fixed linguistic forms (*viz.*, the assumption that in an interactive environment this distance should be of the order of one or two generations of masters and students) support our earlier conclusion that the segregation of a distinct "geometers' *dýnamis*" from a naive-geometric or pebble-based calculators' concept occurred during Plato's youth or shortly before. A central role could then perhaps be ascribed to Hippocrates and Theodoros.

An observation made by Neuenschwander [1973, 329ff] may indicate in which connection the innovation took place. Time and again, the early books of the *Elements* use a principle which is neither proved nor stated as an axiom, *viz.*,

$$AB = CD \qquad <=> \qquad (AB)^2 = (CD)^2$$

Now, it follows from Neuenschwander's analysis that when this principle is applied in Books II and IV, it is most often stated explicitly. When it is used in Books I and III, however, it remains implicit, except in III.35-36; precisely these two propositions deal with areas of parallelograms, and their subject-matter is thus related to that of Book II. We may conclude that only the tradition behind Books II and IV, the "metrical tradition" dealing centrally with areas of plane figures and continuing itself in the theory of irrationals, based intself on a set of concepts making it natural to notice and formulate the application of the principle, which is nothing but the interchangeability of equality *mékei* and *dynámei*. This agrees perfectly with the hypothesis of a Near Eastern borrowing, because the branch of geometry which could be inspired by Babylonian "naive-geometric" algebra (or a Greek "calculators' algebra", for that matter) is precisely the so-called "geometric algebra" of *Elements* II (I shall not mix up the discussion of this much-debated term with the present investigation). It also fits well with the branches of geometry which later make use of the *dýnamis* idiom: *Elements* X and XIII, etc.

A final observation concerns the very idea of a "conceptual import". Truly, the translation of *dýnamis* into *mitḫartum* makes good sense of all occurrences of the term prior to Pappos. Still, the "geometers' *dýnamis*" belongs within a conceptual context differing fundamentally from that of the *mitḫartum*; from the principle that the concepts of a connected body of thought are themselves connected we should therefore expect that the idea of a translation can only be approximately true.

This is in fact borne out by closer analysis of some of our Greek texts. In the definition of "commensurability *dynámei*" in *Elements* X, the entities which are explicitly measured by an area (χῶρος) are the tetragons on the lines. Implicitly, however, the expression supposes that the *lines* regarded in their aspect of *dynámeis* are measured (since the lines themselves are com-mensurable in that aspect). Earlier, in the Eudemos fragment, bases and diameters themselves are said explicitly to have a ratio (*viz.*, the ratio of the areas of their squares) under the same condition. This must mean that the area belonging with a line regarded as parametrization of a square figure is less of an external accessory than the area of a Babylonian *mitḫartum* – the Greeks, apprehending the tetragon-square as well as circles and other plane figures as identical *with their areas*, tended to assimilate the *dýnamis*-square into the same pattern[22]. In the case of the "calculators' *dýnamis*" this becomes even more evident, since the Diophantine *dýnamis* has assumed the numerical role in his problems which the area (a-šà or *eqlum*) and not the *mitḫartum* assumes in Babylonian texts.

Precisely this conceptual incongruity is probably the reason for the disappearance of the terms *dýnamis* and *dýnasthai* from the active vocabulary of geometers by the early third century, except in specific technical niches (commensurability *dynámei*) and formulaic expressions. The terms did not fit the mental organization of Greek mathematics once its various branches and disciplines had gone into the melting-pot of Alexandrian learning.

As to the term *dýnamis* itself, it is clear that the connotational similarity to the *mitḫartum* does not reflect a Babylonian understanding of the square as a result of a confrontation of equals or counterparts. If not accidental, the shared connotations (involving physical force and commercial value) will have to be explained at the level of the "folk etymology" (the "folk" in question being calculators or possibly geometers): as an attempt to understand why the Semitic masters called a "line regarded under the aspect of the appurtenant square" by a strange name related to the confrontation of values and force, an attempt then reflected in the Greek term chosen to denote the same object.

Such a pseudo-etymology may from the beginning have been connected to explanations proposed on the basis of the Greek language: The square which a line "has the power to form", "is worth" or "masters". Such metaphors may also have been introduced as secondary explanation when memory of a foreign origin had been forgotten (which could have happened quickly). A "Babylonian" and a "Greek" interpretation of the term need not be mutually exclusive; in some way they probably supplement each other.

CONCLUSIONS

As stated by Berggren [1984, 402], there are in the early history of Greek mathematics "sufficient documents to support a variety of reconstructions but an insufficient number to narrow the list of contending theories to one". This pessimism is confirmed by the impossibility of reaching consensus on the merits of such great reconstructions as [Szabó 1969] and [Knorr 1975][23]. For the time being, no compelling reconstruction can apparently be written; instead, further progress may be made through construction of scenarios for all or parts of the development, which may open our eyes to hitherto unnoticed features in the source material at hand. Such scenarios should be internally coherent and in agreement with available documents,

[22] Conversely, in its exact form the Greek concept could of course have no place with the Babylonians. A Babylonian line (and any other geometrical entity) is identified by, and conceptually not distinguished from its measuring number. A Greek line, however, is conceptually distinct both from the number of unit lengths contained in it when regarded as a length and from the number of unit squares covering it when regarded *dynámei*.

[23] Cf. also the review of a number of ongoing controversies in [Berggren 1984].

and should be compared with rival interpretations of history on their merits in these respects; however, they need not claim in advance to be necessary truths.

The above discussion, which includes an abundance of hypothetical formulatiosn, is meant primarily to provide suggestions for such a partial scenario. Still, the knitting is not so tight that all parts of the argument stand and fall together; nor are they equally hypothetical.

Among the positively supported results is the distinction between a “geometers’ *dýnamis*” and a “calculators’ *dýnamis*”. Both groups made use of the term, but they did so for different purposes and inside different conceptual frameworks, and hence necessarily in partially different ways – *vide* the quotations from Hero. Direct evidence was also given for the assignment of the crystallization of the geometrical *dýnamis* usage to Plato’s late years – and hence also for the doubt concerning the Hippocratean origin of the exact formulations in the Eudemos fragment.

The interpretation of the geometrical *dýnamis*-concept as “a square identified by, and hence with, its side” is also supported by the sources regarded as a totality in the sense that the apparent ambiguities in the usage can only be surmounted by an interpretation of this kind. The possibility that such a concept was held is established through the *mitḫartum*-parallel.

More hypothetical are the primacy of the “calculators’ *dýnamis*” over the “geometers *dýnamis*”; the interpretation of the early “calculators’ *dýnamis*” as belonging with a naive-geometric or pebble-based “algebra”; the suggestion that the segregation of a distinct “geometers’ *dýnamis*” is connected with the beginnings of the theoretical tradition behind *Elements* II in the later fifth century; and the hypothesis that the *dýnamis* is structurally similar to the *mitḫartum* because it is borrowed. Taken singly, these are nothing but possible hypotheses; together, they appear to form a plausible scenario fitting the complete available evidence, including evidence rarely taken into account (e.g., the finer details of Plato’s formulations in their chronology, the hidden presence and absence of the *dynámei/mékei*-relation in *Elements* I-IV, and the peculiar Neopythagorean usage).

Independent but secondary observations are the disappearance of the *dýnamis*-usage and its sole survival in formulaic language (which is no new idea); and the explanation of this process in terms of the incongruity between the “*dýnamis*-square” and the normal Greek conceptualization of squares and other plane figures as identical with the surface covered.

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